# Math & Realia

Workshop on mathematical practices in various sectors of Mesopotamian society

Organised by Cécile Michel and Christine Proust with the SAW Project
Scheduled on June 26\textsuperscript{th}, 2015, 9 am – 5:00 pm, Bern

## Programme

<table>
<thead>
<tr>
<th>Time</th>
<th>Title</th>
<th>Speaker</th>
</tr>
</thead>
<tbody>
<tr>
<td>09:00-9:30</td>
<td>Introduction Weighing Units, Standards and Weight Artefacts during the Old Assyrian Period</td>
<td>Cécile Michel</td>
</tr>
<tr>
<td>09:30-10:00</td>
<td>Equivalencies: the neo-Sumerian administrative evidence</td>
<td>Hagan Brunke</td>
</tr>
<tr>
<td>10:00-10:30</td>
<td>The Size of the Things – Metrological Practices in the Old Babylonian Diyala</td>
<td>Carlos Gonçalves</td>
</tr>
<tr>
<td>11:00-11:30</td>
<td>Procedures for estimating the fields’ and gardens’ surfaces in Pre-Sargonic documentation from Girsu</td>
<td>Camille Lecompte</td>
</tr>
<tr>
<td>11:30-12</td>
<td>The administration of irrigation systems in the Umma province of the Ur III state (2112-2004 B.C.)</td>
<td>Stephanie Rost</td>
</tr>
<tr>
<td>12:00-12:30</td>
<td>Estimating volume: Methods for assessing volume exhibited and suggested in two Old Babylonian tabular administrative texts</td>
<td>Robert Middeke-Conlin</td>
</tr>
<tr>
<td>14:00-14:30</td>
<td>A procedure text on volume calculations: BM 85196</td>
<td>Pierre Chaigneau</td>
</tr>
<tr>
<td>14:30-15:00</td>
<td>The fractions 1\00 and 2\00 in the Šakkanakku period and their genesis</td>
<td>Laurent Colonna d’Istria</td>
</tr>
<tr>
<td>15:00-15:30</td>
<td>A mathematical class at ‘Chantier K’ in Mari</td>
<td>Grégoire Nicolet</td>
</tr>
<tr>
<td>16:00-16:30</td>
<td>Text and Architecture: YBC 5022 and BM 15285 as “Manuals of an Architect”</td>
<td>Natalie N. May</td>
</tr>
<tr>
<td>16:30-17:00</td>
<td>Making equivalent volume, brickage and capacity in Old Babylonian mathematical texts Conclusion</td>
<td>Christine Proust</td>
</tr>
</tbody>
</table>
Presentation - http://rai.unibe.ch/gb/workshop010/

This workshop is dedicated to the memory of Aizik Abramovich Vaiman (1922-2013)

The workshop "Math & Realia" aims to present researches developed in connection to the SAW project (Mathematical Sciences in the Ancient World, see http://sawerc.hypotheses.org/). Case studies that show how mathematical knowledge was produced, transmitted or used in specific sectors of activity, such as administration, trade, or education, will be developed. More precisely, in the line of the RAI 61, the workshop will focus on the relationship between the texts and their physical environment or mental representations. The topics addressed include the following:

- Fields and lands: different ways to quantify surfaces, mental representations, arrangement of surface units, diagrams.
- Canals, bricks and walls: different ways of quantifying spatial extensions, quantifying labor and organizing work.
- Measures and standards: relationships between units of capacity and standard vessels, units of weight and standard stone weights, textual traces and archaeological traces.
- Trade and exchanges: the mathematical tools to "make equivalent" different kinds of goods, market rates, theoretical rates and bureaucratic rates, regular and non-regular numbers, approximations.
- The walls of schools: textual data and archaeological data.
- Cuneiform signs as images: notations of integers, fractions, and measurement units; arithmograms, metrograms and arithmo-metrograms.

Abstracts

Hagan Brunke, Freie Universität Berlin

Equivalencies: the neo-Sumerian administrative evidence

The mathematical concept of making entities of different categories comparable by relating them to a common reference entity reflects vastly in the neo-Sumerian (Ur III period, 21st century B.C.) administrative record. There are value equivalencies relating several kinds of goods to their primary products, workload equivalencies relating labour to time, and prices in shape of silver value equivalents. Whereas the latter vary over time, the former are time-independent constants, some of which— in the case of workloads— can also be found in the Old-Babylonian coefficient lists. We also find chains of subsequent such conversions, and in one case even „sexagesimally convenient“ approximations for the awkward fraction resulting this way.

Pierre Chaigneau

A procedure text on volume calculations: BM 85196

The text of the tablet BM 85196, dated from the Old-Babylonian period, contains 18 mathematical procedures, most of them being related to the calculation of volume. This text was edited successively by François Thurou-Dangin and Otto Neugebauer during the second half of the 1930's. Apart from 2 problems, the text has not been studied again since then. However, much remains to be said about the related mathematical practices, especially when one takes into account the current knowledge of the relationship between this kind of text and the metrological tables. What is at stake in the different kind of signs used to express numbers and quantities? How this text testifies for the relationship between its procedures and mental representation of volumes? What about the diagrams found in two procedures?
Laurent Colonna d’Istria

The fractions 1/3 and 2/3 in the Šakkanakku period and their genesis

This paper will present new graphemes for the fractions 1/3 and 2/3 attested in texts dated to the end of the Šakkanakku period. These graphemes for the fraction 1/3 and 2/3 find no parallel in contemporary Mesopotamian traditions. Were these graphemes an innovation or a legacy from an older tradition? As the graphic tradition of the end of the šakkanakku period has many archaic features, our purpose is to examine the graphemes for these fractions in the earlier traditions of the third millennium (Early Dynastic and Old Akkadian Periods) from northern and southern Mesopotamia.

Carlos Gonçalves

The Size of the Things – Metrological Practices in the Old Babylonian Diyala

This paper studies measurement values, abstract numbers and calculations in the scope of Old Babylonian mathematical tablets from the region of the Diyala. The first aim is to provide a description of the measurement units used in these tablets and to offer a classification of the manners in which scribes employed them. The paper also presents a new interpretation for the issue of the ‘orders of magnitude’ in cuneiform mathematics, by analysing how the structure of metrological systems was involved in calculations.

Camille Lecompte, CNRS, Paris

Procedures for estimating the fields’ and gardens’ surfaces in Pre-Sargonic documentation from Girsu

The Pre-Sargonic documentation from Girsu sheds some light on the mathematical procedures applied by land surveyors in order to estimate the fields’ and gardens’ surfaces. Approximately fifteen administrative tablets refer indeed to the length, width and surfaces of several fields, which are all superior to one iku (0,36 ha). On the other hand, a couple of texts also give the length, width and surface of gardens or small surfaces calculated in sar units. Both kinds of tablets enable us therefore to reconstruct the relationships between length and surface units, as well as to identify the bridge used by surveyors to shift from the former to the latter. They also show that surveyors used several kinds of rounding and approximations, probably in order to reduce the margin of error. The aim of the paper is first to focus on the methods of the Sumerian land-surveyors to estimate surface in administrative contexts, especially to compare the calculation of large and small surfaces. Furthermore, the fresh review of the use of length units, especially the gi and the ninda-DU, such as they occur in the administrative documentation, can also show which specific unit was considered to be a basic unit.

Cécile Michel

Weighing Units, Standards and Weight Artefacts during the Old Assyrian Period

The private archives of the Assyrian merchants found at Kaneš, in central Anatolia, highlight the use of the weight system, well adapted to quantify the metals they traded from East to West and back. An atypical progressive metrological list of weights gives an idea of the variety of weights used by the merchants. Account texts reveal the use of different weight standards and methods to correct weight deviations. A hundred and fifty stone weights found at Kaneš may be compared with the data provided by the texts. These weights, which show significant variations, witness the existence of many individual weight standards, and confirm the recurrent necessity to correct weight deviations.
Natalie N. May

Text and Architecture: YBC 5022 and BM 15285 as “Manuals of an Architect”

Šitimgallu, a Sumerian loan word for the Akkadian “architect,” “chief” or “master builder” first appears and is used in the common language in fact only in the Old Babylonian period. In all periods of human history, the profession of an architect demanded wide range of knowledge and abilities: inspired vision of an artist and an expertise in various materials, advanced knowledge of mathematics and organisational skills. Are there cuneiform texts that can be associated with this group of experts? Analysing the list of coefficients YBC 5022 and the famous tablet of geometrical problems BM 15285 against the background of archaeological and pictorial evidence, I will claim that these texts served as an “architect’s manual.”

It had been already suggested, based on textual and mathematical grounds, that the origins of Babylonian mathematics go back to the Ur III period. I will argue that the profession of an architect, which involves proficiency in exact science and other expert knowledge, also emerged in this time.

Robert Middeke-Conlin

Estimating volume: Methods for assessing volume exhibited and suggested in two Old Babylonian tabular administrative texts

This presentation attempts to examine methods used to assess volume in an Old Babylonian administrative environment. Texts like NBC 11509, an undated tabular administrative text, appear to estimate the volume of silt to be excavated from canals using a simple multiplication of length by width by depth produces volume. Each of the six canal sections listed on NBC 11509 bears this out. However, when we look at similar texts, such as NBC 6763 dated to Rîm-Sîn of Larsa’s 38th year in power and possibly produced in a similar bureaucratic setting to NBC 11509, this is not the case. On NBC 6763, only one volume measurement value out of five appears to be the product of a multiplication of length by width by depth. Moreover, the total of volume measured is not the sum of volume measurement values stated on the text. This presentation asks “why do expected and stated measurement values on NBC 6763 differ so much?” and “What is the relationship between NBC 6763 and NBC 11509? In answering these questions, it will explore the calculation techniques used to produce each text as well as the administrative environments in which these texts were compiled.

Grégoire Nicolet

A mathematical class at ‘Chantier K’ in Mari

In 1998, J.-C. Margueron discovered at ‘Chantier K’, outside the palace of Zimri-Lim, a large amount of school tablets. Among the word lists, proverbs and literary texts, there were also around 190 mathematical texts: metrological lists and tables, multiplication tables, even tables of squares and square roots, as well as around 50 examples of ‘mathematical rough work’ on buns. As the 61st RAI wants to promote bridges between archeologists and assyriologists, it will be shown how a careful recording of the tablets during the excavation allows us to follow the learning of mathematics thanks to dated mathematical tablets discovered together. Moreover, the metrological tradition highlights a discrepancy between the teaching that took place at Chantier K and the administrative practices of the palace. Finally, the important role played by the learning of mathematics at ‘Chantier K’ confirms the advanced numeracy of the scribes of Mari.
Christine Proust
Making equivalent volume, brickage and capacity in Old Babylonian mathematical texts
How to quantify three-dimensional entities? Ur III and Old Babylonian cuneiform texts reflect various approaches to this issue. Different metrologies, for capacity, volume or brickage, were adopted in different fields of activity for different purposes. Then, the idea of converting one into another, for example, capacity into brickage, was of little practical interest. In Old Babylonian mathematical texts too, one finds these different ways of quantifying three-dimensional entities. However, these approaches to spatial extension are not completely independent from each other. In a way, the idea of converting capacity into volume, or volume into brickage, appeared to be meaningful from a mathematical point of view. How are the capacity, volume and brickage metrologies articulated in mathematical texts? This issue is examined through some mathematical texts from Southern Mesopotamia.

Stephanie Rost
The administration of irrigation systems in the Umma province of the Ur III state (2112-2004 B.C.)
Managing an irrigation system, regardless of time and place, requires a considerable amount of planning. The irrigation infrastructure of canals and water control facilities must first be constructed, and then later regularly maintained. Water needs to be allocated to different parts of the system in order to meet the water demands of various crops. The smooth operation of the system depends on the timely performance of all necessary tasks, and a staff of full and/or part-time administrators is employed to plan and schedule the execution of the required work projects. The administrative documents from the Umma province during the Ur III period (2112-2004 BCE) provide an extraordinarily detailed insight into the administration of ancient irrigation systems and the various computations and quantifications employed in the planning of the execution of irrigation works. These documents bear witness to the level of scientific understanding of data acquisition and management in Ur III administration. In addition, the recording of the names of officials in charge of supervising and authorizing the various work projects not only allow us to reconstruct the chain of command but also determine the degree of centralized control in irrigation management. More importantly however, it also provides insight into the skill set that various administrators and officials mastered and helps us gage the level of literacy and numeracy of an ancient society. This paper will discuss a set of texts which illustrates the individual administrative steps including the mathematical procedures involved in the planning and execution of irrigation works.