

SAW (MATHEMATICAL SCIENCES IN THE ANCIENT WORLD)
NEW THEORETICAL APPROACHES TO THE SOURCES
AND SOCIO-POLITICAL ISSUES OF THE PRESENT DAY



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Summary

The SAW project is devoted to mathematical sources that have come down to us from the ancient world and, more specifically, though not exclusively, to the sources produced in Mesopotamia, China, and the Indian subcontinent. The ambition of SAW is to develop new theoretical approaches in the field of the history of ancient mathematics in order to highlight a motley of practices within what at the present day are too often presented as homogeneous wholes, that is, “Mesopotamian mathematics”, “Chinese mathematics”, and “Indian mathematics.” To this end, SAW intends to concentrate systematically on the mathematical sources produced in relation to two sectors of activity in the ancient world: the practice of the astral sciences and the activities of state administrations in charge of financial matters. The goal SAW has set itself is to shape resources so as to promote new representations of mathematics and its history. The wider societal impact at which the project aims is to define an alternative to widespread visions of mathematics that have made it possible for this field and its history to become tools playing a part in the shaping of communities worldwide. SAW will work to make the outcome of the research available to secondary school teachers, in order to contribute to bridge the widening gap between young people and mathematical sciences.

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I. DISSATISFACTIONS WITH PRESENT-DAY HISTORIOGRAPHY OF ANCIENT MATHEMATICS

1. History of mathematics within history of science

SAW was formulated in response to an analysis of the present-day state of the history of ancient mathematics. The field of the history of the mathematical sciences in the ancient world is diminishing in size, attracting fewer and fewer young scholars, especially on topics that do not deal with Europe. It is becoming more and more marginalized in the history of science as well as in the history of mathematics. The field is cut into two because of the impact of present-day disciplinary boundaries (the history of mathematics being severed from that of astronomy). Moreover, the field of the history of mathematical sciences in the ancient world is further divided into two trends. One part of it—a small one—focuses on cultural and social history without attending to the details of mathematical ideas and concepts. This is the part of the field that is best inserted in the wider history of science. The other part stresses mathematical or astronomical concepts and results, without attending to their cultural and social dimensions. Ironically, despite their differences, both approaches display a picture of mathematics as if its nature is and has always been unchanging and as if its content had been homogeneous throughout the centuries.

All these problems are compounded in the case of the history of what is regularly called by the strange expression “non-Western mathematics.” This fact has resulted in a dramatic situation, since this state of the field leaves standard views that considered Europe as having played the major part in the history of ancient mathematics in their dominant positions.

The SAW project was conceived as a response to this challenge. It concentrates on mathematical sources from the ancient world, more specifically, from Mesopotamia, China, and the Indian subcontinent. Its first goal is to strengthen research in these subfields, both by bringing young scholars to them and by addressing new issues and developing new methods. The ambition is to run counter to all the trends described above and renew the approach to these sources from a theoretical point of view, thereby turning the subfield of the history of mathematical sciences in the ancient world into a significant source of innovation for the whole discipline. The decision to concentrate mainly on Mesopotamia, China and the Indian subcontinent derives from the principal theoretical objective SAW set itself. This objective was defined by taking into account the wider societal impact that the history of mathematics exerts in our societies worldwide.

2. POLITICAL AND SOCIAL USES OF THE HISTORY OF ANCIENT MATHEMATICS

The SAW project also derives from an analysis of how the field of ancient mathematics is loaded at the present time with social and political issues. In the 19th century, the institution of nation-states played a key part in the formation of the discipline of history of science. This relates to an essential fact: due to the prestige attached to science, and also specifically to mathematics, history of science has provided some of the main cultural artefacts that entered into the making of nations. This also held true for empires, “civilizations,” or other kinds of “communities.” Means have been invented to create communities around science and its history. SAW intends to analyze *in detail* how the connection between the making of communities and the historiography of science was shaped and how the process is still at work in the modern world. Eurocentrism in the history of mathematics was, at least partly, a product of this phenomenon. This factor has also, most probably, played an important part in maintaining Eurocentrism in its dominant historiographic position until today. All over the world, at the present time, communities claim their value on account of “their contributions” to mathematics, unless they claim their difference on account of the “specific kind of mathematics” that is suited to them.

The main objective SAW set itself was determined by an observation. The historiographies that are affected by these phenomena present mathematics as a discipline that shows an overwhelming uniformity. This uniformity is approached in two ways. Either the nature of mathematics is assumed not to have changed throughout history —this is what we shall call “global uniformity”—, and the various communities are taken to be characterized by their “firsts”, or mathematics is portrayed as being practised in a specific way, depending on the nation or the “civilization” considered (“the West,” China, India, Mesopotamia, and so on). Mathematics would present various brands of “regional uniformity.” One would thus be entitled to speak of “Western mathematics” or “Chinese” or “Indian,” mathematics.

Let us leave aside the concepts of “precursor,” or “first,” whose use has already been analyzed in depth, and concentrate on these assumptions of “uniformity.” The former conception of the “uniformity” of mathematics —the “global uniformity”—evokes a faith in the uniformity of the scientific practice, epitomized by the belief in a “single” scientific method. In the last decades, much effort has been devoted, in history and philosophy of science, to expose the shortcomings of such a view. This effort has borne more on science in general than on mathematics in particular. More work remains to be done more systematically on mathematics along the same lines. Interestingly enough, historians like Richard Yeo

have argued that this emphasis on “the” scientific method, which is so perceptible among “scientists” from the 19th century onwards, had to be correlated to the shaping of “the scientific community” after 1800. One may thus interpret the effort that historians and philosophers of science recently placed in this direction as a critical analysis of an idea of scientists as forming a “community.”

Much less work, however, has been devoted to the latter conception of the “uniformity” of mathematics—the “regional uniformity”. And yet, it is the one that has probably the tightest connection to the making of “communitarian” historiographies, which can be identified at work in some trends in all the societies of our global world. It was this observation that determined where SAW wanted to exert its theoretical effort. Its main goal is to expose the limitations of views that consider the mathematics produced in either Mesopotamia, or India, or else China, as “uniform.” The project aims to highlight *how* one can identify, in these three geographical areas, different mathematical practices and bodies of knowledge, anchored in specific social and professional contexts. At a theoretical level, the fulfilment of this objective will require a critical reflection on the concept of a “culture of scientific practice”—to be distinguished from more essentializing uses of the word “culture.” Moreover, we shall have to address the issue of how different cultures, in a sense to be determined, connect with each other—we will rather suggest “overlap” with each other. This question is fundamental, if we are to take as far as possible our critical analysis of representations in terms of community. However, the question is no less fundamental in general history and philosophy of science at the present time, since we find ourselves at a juncture where we need theoretical tools that will allow us to move from the description of a variety of local practices to a more global picture.

II. AIMS OF SAW

The objective is to introduce an initial decomposition into smaller entities of what at the present day is too often presented as homogeneous wholes, that is, “Chinese mathematics,” “Indian mathematics,” and even “Mesopotamian mathematics.” The strategy adopted to do so is systematic. Whether we look at Sumero-Akkadian, Sanskrit, or Chinese sources documenting ancient mathematical practices, in each area, some of the sources adhere to spheres of astronomical/astrological activity, whereas others appear to be closer to the workings of an administration dealing with financial matters. It is because the three sets of sources that were handed down from Mesopotamia, China and the Indian subcontinent present that same feature that we have chosen to concentrate on these three areas in parallel for the SAW project. The project intends to focus on sources specifically adhering to either domain of activity and to identify, by contrasting them with each other, specificities in the practice of, and knowledge in, mathematics to which they bear witness. Our expertise in these subfields is sufficient to know that the method will yield results. These sources represent the material means with which SAW will realize its program.

Even though mathematical knowledge and practice appear to differ from social context to social context in ways that need to be understood, they also partly overlap. This remark may explain why previous historical research has failed to identify these patterns of differences and has offered a vision of “cultural” uniformity. This feature is quite important on a theoretical level. We are indeed confronted

with local cultures of mathematical practice that are not confined within sealed boundaries, but share common knowledge and ways of working. SAW aims at finding the theoretical tools to describe this pattern of dissimilarity and overlap in greater detail. This is the conceptual breakthrough for history of science on which SAW will work, —its theoretical means. The situations observed already appear to be promising in this respect and yield sufficient empirical evidence to serve as a basis for this exploration.

In addition to work on primary sources, SAW intends to carry out a reflection on the history of historiography of mathematics. Such a reflection is essential to approach our sources in a critical way. The main focus of SAW in that direction will be on the key general operations which are at play in the making of the historiography of ancient science: the shaping of critical editions, the constitution of collections, the conceptual tools used to make the past in the present.

III. THEORETICAL TOOLS

On a theoretical level, SAW does not start from scratch; we can rely on a considerable amount of research work. Contemporary research in the history and philosophy of science has placed much emphasis on shaping conceptual tools with which one can approach, from different perspectives, various features of “local” scientific practices. It is impossible to do justice to the innumerable suggestions that have been put forward. Let us mention three examples to show where we situate ourselves in this landscape.

Following A. Crombie’s conception, which was expounded in its fullest form in *Styles of scientific thinking in the European tradition* (1994), I. Hacking has elaborated his concept of “style of reasoning” (for instance, Hacking 1992). Hacking identifies six styles, depending on the “method” adopted for the inquiry (axiomatico-deductive reasoning, laboratory style and so on). His focus lies on understanding the main components that need to be taken into account to capture all the dimensions of a style. Hacking thus suggests a systematic tool to approach the variety of scientific inquiries. However, this variety is considerably reduced for mathematics, for which Hacking only considers “axiomatico-deductive reasoning.” This conception can be challenged if we consider a much wider set of sources. Moreover, Hacking’s goal has the counterpart that he does not attend to the historical transformation of his styles. In our view, they can be a tool for our endeavour, but by no means the only one. SAW will have suggestions to make to refine this tool.

In her book *Epistemic cultures*, K. Knorr-Cetina (1999) attempts to identify, within given disciplines, short-term configurations that include collectives and their modes of organisation, knowledge machineries and their uses, her main aim being to account for “how we know what we know.” Several of her suggestions can inspire an approach to ancient mathematics, for instance, the analysis of “knowledge machineries.” However, the difference between our objects —present-day activity versus ancient texts— and our modes of access to the object of study —ethnography versus the analysis of documents— limit the possibility of a full transfer. The ethnographic approach adopted allows her to deal with short-term phenomena, which are out of reach with ancient documentations. Moreover, the

nature of the human collectives engaged in the inquiry remains most of the time unknown, at least in its details. The full scope of her approach is intrinsically limited to the observation of scientific activity at the present time.

In *Making sense of life*, Evelyn Fox Keller (2002) introduces a concept of “epistemological culture,” from which again we can derive inspiration. Her emphasis is placed on the fact that in order to describe the working of a collective engaged in scientific inquiry, it does not suffice to attend to the “local practices —the techniques, the instruments, and the experimental systems—of a particular scientific subculture.” In other words, the features usually described by historians are not rich enough to capture broadly the specificities of how knowledge activities are conducted within a collective. In her view, one must also take into account “the norms and mores of a particular group of scientists that underlie the particular meanings they give to words like theory, knowledge, explanation and understanding, and even to the concept of practice itself.” These meanings, she emphasizes, betray an unexpected variety. As a result, the problems that the practitioners operating within a given collective will find important to address and the kinds of answer they will consider appropriate will differ, depending on key epistemological choices shared by the collective. Therefore, the knowledge produced will bear the hallmark of this global set of factors.

These conceptual attempts at providing means for approaching in a systematic way the diversity of scientific practice were all mainly designed while dealing with modern science —at the earliest, early modern science. They overwhelmingly consider only European or North-American source material. Mathematics plays a minor part, if a part at all, in all three. However, they all provide insights to shape analytical tools that we shall need in order to be attentive to all the details needed in our sources and perceive differences in mathematical practice.

In the last years, and through an engagement with ancient Chinese sources, K. Chemla has shaped a first set of tools suited to approach mathematical practice along these lines. She has suggested to decompose the knowledge machineries into “elements” (problems, algorithms, diagrams, kinds of text and inscriptions used, and so on) in order to gather evidence allowing the historian to restore how actors worked with this kind of elements. This is what she calls “practices with elements.” In this respect, as E. Fox Keller had suggested, epistemological factors proved to be essential: what epistemological values did actors prize? What were the objectives actors assigned to proof? Raising these kinds of questions enables a description of the nexus of practices with elements, into which a given way of doing mathematics must be analyzed. Such an analysis is crucial to compare and distinguish between different ways of doing mathematics. Moreover, it is also essential to introduce the “practices” that actors have shaped with these “epistemological elements.” In fact, their epistemological practices permeate the practices with other material elements. For example, in ancient China, the way generality is practiced echoes into how problems and algorithms are written and as well as how diagrams are designed. This whole set of practices shapes a network of relationships. K. Chemla suggests that the expression of “culture of mathematical practice” refers to such a set. These tools are discriminating enough to approach more extensively different corpuses of ancient mathematical sources worldwide and identify different practices. This is what the SAW project plans to do. We believe that the descriptive power of the tools should now allow us to identify differences in practice and body of knowledge within “Chinese mathematics,” “Indian mathematics,” and “Mesopotamian mathematics.”

However, SAW will not limit itself to apply analytical tools that were already designed. In addition to further refining these tools through extending their application, we have another theoretical perspective in mind. As was indicated, if we rely on Chinese sources, we can already perceive that the extant mathematical writings composed within the context of a practice of astral sciences and those which were used by civil servants in charge of administrative matters present differences as well as a given amount of overlap. They do not form cultures that would need to be identified as separate entities, before raising the question of their exchanges, for instance, through “trading zones.” Rather, these cultures appear to belong to a continuum with specific poles and common regions. This is the picture for which we plan to explore new conceptual tools, in an attempt to establish an approach to “cultures without culturalism.” We are convinced that these phenomena can be analyzed in a precise way, with the tools we have started to design. We should thus be able to describe the pattern of similarity and overlap with a fair degree of precision.

IV. GENERAL AXES OF RESEARCH

1. The definition and use of measuring units —The practice of problems

One of the key operations needed by any administration in charge of financial matters is the definition of measuring units. This operation requires mathematical work for which we have evidence in our sources. This work has scarcely been studied. Much more remains to be done. This aspect is quite meaningful for the SAW project. Once measuring units were defined, the way in which they were used and the way in which practitioners operated with the results of such measures are key features in the identification of distinct mathematical cultures. Moreover, mathematical sources provide evidence that complements aspects on which administrative sources remain silent. We think, in particular, of the measure of amounts of grains, extensions of fields and capacities of granaries that were so essential to the management of the state. The astral sciences also required measuring units. The comparison between the systems of measuring units used by our various sources, the connection that can be established between them on this basis, and the evidence on how one operated on quantities in mathematical practice will lie at the focus of our first workshop. The mathematical documents adhering to the working of the administration seem to have used problems more often. Special attention will be devoted to the use of mathematical problems and to whether they manifest or not adherence to an administrative practice.

2. Astronomy and calendar—The practice of tasks

In addition to paying attention to the practice of mathematics attached to the description and determination of astronomical phenomena — the motion of the planets, computations of specific moments such as eclipses, risings and settings — we plan to focus on how mathematics was involved in the production and use of calendars and almanacs. In relation to these issues, we shall examine who were the people in charge of producing these artefacts. Who used calendars and almanacs, and what knowledge was required for their use? The mathematical documents adhering to astral practices seem to have more often used tasks to introduce mathematical procedures. We will test this hypothesis.

3. History and historiography of mathematical cultures

On the basis of the research carried out during the first three years, we shall attempt to work out an initial synthesis on the identification of various mathematical practices in our three geographical areas. We shall consider the pattern of differences and overlap between the various local cultures identified. The aim will be to suggest theoretical tools in this respect that can be used in history of science and beyond. In addition, we shall examine the uses of histories of past mathematics within various political arenas. Moreover, we shall concentrate on examining in a critical way the ways in which past historiographies have described “communitarian” mathematics as homogeneous. Several features will have to be examined in this respect, such as the history of critical editions and collections of sources, and the modern writing of histories. We shall elaborate the theoretical tools needed to offer the first elements of a new historiography for ancient mathematics. At this point, we will prepare source material meant for a broader audience and especially for high-school teachers to promote a new vision of ancient mathematics.

V. SPECIFIC RESEARCH PROGRAMS

1. Mathematical sciences in Mesopotamia

Cuneiform sources provide us with a wealth of information on mathematical sciences and measuring practices within different milieus, at different time periods, ranging between the early third millennium and the end of the first millennium B.C.E.. This feature is quite important for the project, since it allows us to identify distinct mathematical cultures, depending on the activity in relation to which mathematics was carried out, before we can raise the question of the contact zones and circulation between them. With respect to cuneiform sources, the project will concentrate on three kinds of issues.

First, we shall address the issue of the uniformity of mathematical practices in several ways: do schools of different cities manifest a uniformity of practice? Do administrations display the same practices as schools, and in cases when connections can be brought to light, are there schools that are closer to administrative practices? The reason why the comparison is to be done in this way is that there are indications showing that professional milieus, such as that of scribes, travelled and exchanged practices among themselves more than with the milieus in administration.

Secondly, we shall analyze in detail the mathematical practice developed in relation to astronomical activity. We shall focus only on one of the two archives essential in this respect: Uruk in the first millennium B.C.E.. Its archives contain tablets that are purely mathematical as well as tablets devoted to astral topics. The question will be to compare the mathematical practices in both corpuses. SAW plans to publish editions of these sources and to describe the specific mathematical culture to which they attest.

Thirdly, a key feature of the project is to shape methods that ground our interpretation of ancient sources in the critical awareness that provides a material and social history of the archives and collections of sources. Several directions of research will be explored in this respect.

2. Mathematical sciences in China

First, several sources attest to the mathematical practice in milieus dealing with the astral sciences. They were never approached from this perspective. We shall rely on a commentary to one of these writings, since such writings are essential to find evidence about actual practice. This is what K. Chemla's work on the commentaries of *The Nine Chapters* brought to light.

Secondly, three newly discovered documents were composed within the context of milieus in charge of an administration dealing with financial matters. The first one has already been published and translated. However, there is much to be done to describe the mathematical practice to which it bears witness and to contrast it with what will be found about astronomy. Two other discovered manuscripts should be published soon. In this same vein, we shall study the sources that inform us about the mathematical work engaged in the making of measure standards and their history.

The biography of a key figure at the intersection of these two worlds, Li Chunfeng, will be the focus of one scholarship awarded. Such an analysis is essential as a preliminary work on the history of sources that were handed down, since Li Chunfeng played a key part in how much of our source material survived. Several other actions will be organized to analyze from different perspectives the forces at work in shaping the remaining evidence about mathematical activities in ancient China.

3. Mathematical sciences in India

First, some key writings about mathematics were published as chapters in astronomical treatises. What kind of mathematics do we find here? What are its connections with the astronomy expounded in the same treatises? How do commentators connect the two parts of the treatise? These questions seem quite natural. However, they have never been addressed systematically. SAW will focus on the practice of mathematics that took shape in relation to the pursuit of astronomy in this scholarly tradition.

Secondly, other books that were handed down in the Indian subcontinent belong to a tradition of "practical mathematics." We shall consider whether one can identify through them milieus that have been practicing mathematics in the Indian subcontinent and types of mathematical cultures, different from those described in relation to astronomy. As in the previous parts of the project, we shall rely on the definition and use of measuring units to find precise social contexts for the production of these texts and inquire into their connection to administrative matters.

Thirdly, the history of the historiography of mathematics in India and the analyses of historiographies in different political contexts will be studied through several precise case studies.

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